Małgorzata Sobota, biotechnologia, EXTRA HOMEWORK 15.01.2010

a) $(x^2\sin x + x\sin^2 x + \sin^3 x)' =$

First we need to know is that derivative of sum is sum of derivatives, so we can write:

$= \underset{*}{(x^2\sin x)'} + \underset{**}{(x\sin^2 x)'} + \underset{***}{(\sin^3 x)'} \ominus$

* In the first bracket there is a product, so to find the derivative we must use the formula for derivative of a product.

$(x^2\sin x)' = (x^2)'\sin x + x^2(\sin x)' = 2x\cdot\sin x + x^2\cos x$

Derivative of product: x square and the sine function.

According to the formula it would be product of: the derivative of x square and sine plus x square and the derivative of sine

It would be product of two x and the sine plus the product of x square and the cosine function

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In the second bracket there is also a product, this time of x and sine square. We must use the formula for the derivative of product

$(x\sin^2 x)' = (x)'\sin^2 x + x(\sin^2 x)' =$

Now we have the product of the derivative of x and the sine square plus product of: x and the derivative of sine square.

$= 1\cdot\sin^2 x + x\cdot 2\sin x\cdot\cos x$

The derivative of x it's one, so we have the sine square plus the product of x and two sine and cosine, which is the derivative from sine square function. (The most outer function is the square function α^2, so we have to write its derivative: 2α → in this example it's two sine, we must remember about derivative of sine as the derivative of inside. And it is cosine)

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In the third bracket there is sine cubic. So we must remember that the most outer function is cube α^3 and its derivative is $3\alpha^2$, so:

$(\sin^3 x)' = 3\sin^2 x\cdot\cos x$ ← and also cosine as the derivative of the inside

So the whole derivative would be:

$\ominus\ 2x\sin x + x^2\cos x + \sin^2 x + x\,2\sin x\cos x + 3\sin^2 x\cdot\cos x$

b) $\left(\dfrac{\sin x + \cos x}{\operatorname{tg} x}\right)' =$

In this example there is the derivative of a quotient, so we need to use a special formula to calculate this.

It would be:

$$\frac{(\sin x + \cos x)'\operatorname{tg}x - (\sin x + \cos x)(\operatorname{tg}x)'}{\operatorname{tg}^2 x} =$$

It's product of the derivative of sine plus cosine and tangent. Then we must subtract the product of sum of sine and cosine and the derivative of tangent. All this we must divide ~~all of it~~ by tangent square.

Derivative of sum is the sum of derivatives, so

$(\sin x + \cos x)' = (\sin x)' + (\cos x)' = \cos x - \sin x$

Derivative of sine is cosine

Derivative of cosine is minus sine.

Derivative of tangent its : $(tg x)' = \frac{1}{\cos^2 x}$ One by cosine square

We could write tangent as fraction of sine ~~and~~ by cosine

$$\ominus \frac{(\cos x - \sin x)\, tg x - (\sin x + \cos x)\cdot \frac{1}{\cos^2 x}}{tg^2 x} = \frac{\cancel{\cos x}\cdot\frac{\sin x}{\cancel{\cos x}} - \frac{\sin x \cdot \sin x}{\cos x} - \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x}} =$$

$$= \left[\sin x - \frac{\sin^2 x}{\cos x} - \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\cos^2 x}\right]\cdot\frac{\cos^2 x}{\sin^2 x} =$$

Now we must multiply all of ingredients in the bracket by sosine square by sine square.

$$= \frac{\cos^2 x}{\sin x} - \frac{\cos^{\cancel{2}} x}{\cancel{\cos x}} - \frac{1}{\sin x} + \frac{\cos x}{\sin^2 x} =$$

It is cosine square by sine square

minus cosine

minus one by sine

plus cosine by sine square

We can simplify it to:

$$= ctg x \cdot \cos x - \cos x - \frac{1}{\sin x} + \frac{ctg x}{\sin x}$$

Its product of cotangent and cosine

minus cosine

minus one by sine

plus cotangent by sine

c) $\left(\frac{1}{tg^2 x + ctg^2 x}\right)' =$ Derivative of quotient, so we must use the same formula as in the example b,

$$= \frac{(1)'(tg^2 x + ctg^2 x) - 1(tg^2 x + ctg^2 x)'}{(tg^2 x + ctg^2 x)^2} =$$ Derivative ~~of~~ constant is zero, so we have

$$= \frac{-[(tg^2 x)' + (ctg^2 x)']}{(tg^2 x + ctg^2 x)^2} = \frac{-\left[2 tg x\cdot\frac{1}{\cos^2 x} + 2 ctg x\cdot\frac{1}{\sin^2 x}\right]}{tg^4 x + 2\underbrace{tg^2 x\cdot ctg^2 x}_{} + ctg^4 x} = \frac{-\frac{2 tg x}{\cos^2 x} - \frac{2 ctg x}{\sin^2 x}}{tg^4 x + 2 + ctg^4 x} =$$

it is multiplication formula $(a+b)^2 = a^2 + 2ab + b^2$

product of tangent and cotangent is one

To simplify the expression in the numerator we must lead both fractions to the same denomination which would be $\cos^2 x \cdot \sin^2 x$, so we'll have

$$\frac{-2\,\frac{\sin x}{\cos x}\cdot\sin^2 x - 2\,\frac{\cos x}{\sin x}\cdot\cos^2 x}{\cos^2 x\cdot\sin^2 x}$$ and all these divide by $tg^4 x + 2 + ctg^4 x$,

so finally we'll get:

$$= \frac{-2\,\frac{\sin^3 x}{\cos x} - 2\,\frac{\cos^3 x}{\sin x}}{\cos^2 x\cdot\sin^2 x\,(tg^4 x + 2 + ctg^4 x)}$$

Its in the numerator: minus two sine cube by cosine minus two cosine cube by sine and all these by product of cosine square and sine square and sum of tangent to fourth power plus two plus cotangent to fourth power