EIGENVALUES AND EIGENVECTORS

Find: A) eigenvalues and eigenvectors of matrix $A=\begin{bmatrix}1 & 8\\ 2 & 1\end{bmatrix}$, and

B) eigenvalues of the following matrices: A^3, A^{-1}, $7A$, $A-2I$

A)

1) To calculate λ (eigenvalue), we have to follow the formula $|A-\lambda I|=0$

$A=\begin{bmatrix}1 & 8\\ 2 & 1\end{bmatrix}$

$$|A-\lambda I| = \begin{vmatrix}1-\alpha & 8\\ 2 & 1-\alpha\end{vmatrix} = (1-\alpha)^2-16 = 1-2\alpha+\alpha^2-16 = \lambda^2-2\alpha+15=0$$

$\Delta = 4+(4\cdot 15)=64 \quad \sqrt{\Delta}=8$

eigenvalues of matrix $A=\begin{bmatrix}1 & 8\\ 2 & 1\end{bmatrix}$

$\lambda_1 = \frac{2-8}{2} = -3$

$\lambda_2 = \frac{2+8}{2} = 5$

2) Now we have to calculate the eigenvectors of this matrix. To do that, we should create two systems of equations

a) $\underbrace{\begin{bmatrix}1 & 8\\ 2 & 1\end{bmatrix}}_{A}\cdot\begin{bmatrix}a\\ b\end{bmatrix} = \underbrace{-3}_{\lambda_1}\begin{bmatrix}a\\ b\end{bmatrix}$ ← in this step we have to use the theorem of eigenvector: $A\cdot V_\lambda = \lambda\cdot V_\lambda$

$$\begin{cases}a+8b=-3a\\ 2a+b=-3b\end{cases} \Rightarrow \begin{cases}8b=-3a-a\\ 2a=-3b-b\end{cases} \Rightarrow \begin{cases}8b=-4a \,|\!:8\\ 2a=-4b\end{cases} \Rightarrow \begin{cases}b=-\frac{1}{2}a\\ a=-2b\end{cases} \Rightarrow V_{-3}=\begin{bmatrix}-2b\\ b\end{bmatrix} \text{ ex. } \begin{bmatrix}-8\\ 4\end{bmatrix}$$

b) $\underbrace{\begin{bmatrix}1 & 8\\ 2 & 1\end{bmatrix}}_{A}\cdot\begin{bmatrix}c\\ d\end{bmatrix} = \underbrace{5}_{\lambda}\begin{bmatrix}c\\ d\end{bmatrix}$ ← we have to repeat the calculations for λ_2

$$\begin{cases}c+8d=5c\\ 2c+d=5d\end{cases} \Rightarrow \begin{cases}8d=5c-c\\ 2c=5d-d\end{cases} \Rightarrow \begin{cases}8d=4c \,|\!:8\\ 2c=4d \,|\!:2\end{cases} \Rightarrow \begin{cases}d=\frac{1}{2}c\\ c=2d\end{cases} \Rightarrow V_5=\begin{bmatrix}2d\\ d\end{bmatrix}, \text{ ex } \begin{bmatrix}8\\ 4\end{bmatrix}$$

B) The best way to calculate the eigenvalues for following matrices is using the theorem, which says:

A: $\lambda_1, \lambda_2, \dots \lambda_n$
A^n: $\lambda_1^n, \lambda_2^n, \dots \lambda_n^n$
$k\cdot A$: $k\cdot\lambda_1, k\cdot\lambda_2, \dots k\cdot\lambda_n$
$A-kI$: $\lambda_1-k, \lambda_2-k, \dots \lambda_n-k$

Using the theorem we have:

A^3: $\lambda_1=-27 \quad \lambda_2=125$

A^{-1}: $\lambda_1=-\frac{1}{3} \quad \lambda_2=\frac{1}{5}$

$7A$: $\lambda_1=-21 \quad \lambda_2=35$

$A-2I$: $\lambda_1=-5 \quad \lambda_2=3$

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EIGENVALUES AND EIGENVECTORS

Find: A) eigenvalues and eigenvectors of matrix $A=\begin{bmatrix}1 & -3\\ -2 & 2\end{bmatrix}$, and

B) eigenvalues of the following matrices: A^2, $5A$, $A-3I$

A) 1) To calculate the eigenvalue (λ) of matrix A, we have to follow the formula $|A-\lambda I|=0$

$A=\begin{bmatrix}1 & -3\\ -2 & 2\end{bmatrix}$

$$|A-\lambda| = \begin{vmatrix}1-\lambda & -3\\ -2 & 2-\alpha\end{vmatrix} = (1-\lambda)\cdot(2-\alpha)-6 = 2-2\lambda-\lambda+\lambda^2-6 = \lambda^2-3\lambda-4$$

$\Delta = 9+16=25$

$\sqrt{\Delta}=5$

$$\boxed{\begin{aligned}\lambda_1 &= \frac{3-5}{2} = \frac{-2}{2} = -1\\ \lambda_2 &= \frac{3+5}{2} = \frac{8}{2} = 4\end{aligned}}$$

2) Now we have to calculate the eigenvectors of this matrix $[V_{-1}$ and $V_4]$

To do that, we should create two systems of equations

a) $V_{-1}=\begin{bmatrix}a\\ b\end{bmatrix}$

$$\underbrace{\begin{bmatrix}1 & -3\\ -2 & 2\end{bmatrix}}_{A}\cdot\begin{bmatrix}a\\ b\end{bmatrix} = \underbrace{-1}_{\lambda_1}\begin{bmatrix}a\\ b\end{bmatrix}$$

$$\begin{cases}a-3b=-a\\ -2a+2b=-b\end{cases} \Rightarrow \begin{cases}-3b=-a-a\\ -2a=-b-2b\end{cases} \Rightarrow \begin{cases}-3b=-2a\\ -2a=-3b\end{cases} \Rightarrow \begin{cases}b=\frac{2}{3}a\\ a=\frac{3}{2}b\end{cases} \quad V_{-1}=\begin{bmatrix}\frac{3}{2}b\\ b\end{bmatrix} \quad ex\ V_{-1}=\begin{bmatrix}9\\ 6\end{bmatrix}$$

b) $V_4=\begin{bmatrix}c\\ d\end{bmatrix}$

$$\begin{bmatrix}1 & -3\\ -2 & 2\end{bmatrix}\cdot\begin{bmatrix}c\\ d\end{bmatrix} = 4\begin{bmatrix}c\\ d\end{bmatrix}$$

$$\begin{cases}c-3d=4c\\ -2c+2d=4d\end{cases} \Rightarrow \begin{cases}-3d=4c-c\\ -2c=4d-2d\end{cases} \Rightarrow \begin{cases}-3d=3c\\ -2c=2d\end{cases} \Rightarrow \begin{cases}d=-c\\ c=-d\end{cases} \quad V_4=\begin{bmatrix}-d\\ d\end{bmatrix} \quad ex.\ V_4=\begin{bmatrix}-2\\ 2\end{bmatrix}$$

B) The best way to calculate the eigenvalues for the following matrices is using the theorem, which says: $A: \lambda_1, \lambda_2 \ldots \lambda_n$

$A^m: \lambda_1^n, \lambda_2^n \ldots \lambda_n^n$

$k\cdot A: k\cdot\lambda_1, k\cdot\lambda_2 \ldots k\cdot\lambda_n$

$A-kI: \lambda_1-k, \lambda_2-k \ldots \lambda_n-k$

Using the theorem we have:

A^2: $\lambda_1=1$ $\quad\lambda_2=16$

$5A$: $\lambda_1=-5$ $\quad\lambda_2=20$

$A-3I$: $\lambda_1=-4$ $\quad\lambda_2=1$

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