Problems in composite materials

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Part A – Introduction

A.1
Proceed from the rule of mixtures (ROM) and calculate the percentage of load carried by fibres for a high modulus (HM) carbon fibre/Nylon 6.6 UD composite containing 55 vol.% fibres.

A.2
Write an expression for the modulus of elasticity for a hybrid composite in which all fibers of both types are oriented in the same direction. Using this expression, compute the longitudinal modulus of elasticity of a hybrid composite consisting of aramid and glass fibers in volume fractions of 0.30 and 0.40, respectively, within a polyester resin matrix (modulus of elasticity = 2.5 GPa).

A.3
A tubular filament-wound* composite shaft is to be designed that has an outside diameter of 70 mm, an inside diameter of 50 mm and a length of 1.22 m. The mechanical characteristic of prime importance is bending stiffness in terms of the longitudinal modulus of elasticity. Strength and fatigue resistance are not significant parameters for this application when filament composites are utilised. Stiffness is to be specified as maximum allowable deflection in bending when subjected to 3-point bending with a load of 890 N applied in the center is to produce a deflection of no more than 0.33 mm. A circumferential winding pattern is to be used with a winding pitch of $\theta = 15^\circ$ to the length axis of the shaft (see figure below).
This is to maximise the axial stiffness of the shaft. Possible fibre materials are glass, carbon in high strength (HS), intermediate modulus (IM), high modulus (HM) grades and Kevlar™. The matrix material is to be an epoxy resin, and the maximum allowable fibre volume fraction is 0.60. Decide which ones of these materials that meet the maximum deflection requirement and find the alternative with the lowest material cost. Also, compare these materials with steel and aluminum based on the same deflection criteria and comparing the total weight of the shaft. Use the following material data:

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>ELASTIC MODULUS [GPa]</th>
<th>DENSITY [g/cm³]</th>
<th>COST [€/kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass fibres</td>
<td>74</td>
<td>2.58</td>
<td>2.50</td>
</tr>
<tr>
<td>HS carbon</td>
<td>230</td>
<td>1.80</td>
<td>25.00</td>
</tr>
<tr>
<td>IM carbon</td>
<td>285</td>
<td>1.80</td>
<td>55.00</td>
</tr>
<tr>
<td>HM carbon</td>
<td>400</td>
<td>1.80</td>
<td>175.00</td>
</tr>
<tr>
<td>Kevlar™</td>
<td>130</td>
<td>1.45</td>
<td>65.00</td>
</tr>
<tr>
<td>Epoxy resin</td>
<td>2.4</td>
<td>1.14</td>
<td>8.00</td>
</tr>
<tr>
<td>Steel</td>
<td>210</td>
<td>7.8</td>
<td></td>
</tr>
<tr>
<td>Aluminium</td>
<td>70</td>
<td>2.8</td>
<td></td>
</tr>
</tbody>
</table>

(* filament winding is a manufacturing route and will be discussed in Part C)

A.4
A continuous and aligned fibre-reinforced composite is to be produced consisting of 30 vol% aramid fibres in polycarbonate matrix. Mechanical properties are as follows:
modulus of elasticity for aramid fibre = 131 GPa
modulus of elasticity for polycarbonate = 2.4 GPa

Assume that the composite has a cross-sectional area of 320 mm² and is subjected to a longitudinal load of 44500 N.
Calculate:
- The fibre-matrix load ratio
- The actual loads carried by both fibre and matrix
- The magnitude of the stress on each of the fibre and matrix
- What strain is experienced by the composite?
A.5

**Figure**  
Carpet plots representing: (a) the Young’s modulus, and (b) strength, of carbon fibre/epoxy composite (laminate) containing 65 vol% fibre. The stacking sequence is $[0^\circ/\pm45^\circ/90^\circ]$.

Proceed from the carpet plots given above, and design a composite to meet requirements:
- material should be stronger than steel in the x direction ($\sigma_x = 500$ MPa)
- stiffness in the y direction should be equal to that of aluminium ($E_y = 70$ GPa).

A.6
Give two widespread examples of natural composites, two widespread engineering composites used in the civil sector, one composite for sure allowing cars to roll, and one engineering composite you think can be called advanced.

A.7
The tremendous effort in the field of composite materials and the importance of these materials can be better realised by looking at some milestone dates leading to these materials.

A.8
Describe the use of composite materials in the Beechcraft Company’s Starship 1 aircraft, where certain type of coupling occurring in laminates was implemented to build a smart wing.

A.9
Proceed from the Young’s modulus-density chart by Ashby, and find the subset of materials which have the highest merit indices $E/\rho$, $E^{1/2}/\rho$ and $E^{1/3}/\rho$ important for lightweight design of rods, beams/shafts/struts, and plates, respectively.
A.10
Design a most lightweight cylindrical tie rod of specified length and stiffness.

Part B – Reinforcements and matrices

B.1
The Weibull distribution is different from the normal distribution, as shown in figure below

![Normal and Weibull Distributions](image)

Based on above plots, explain the influence of the count (number of fibres in bundles) on bundle strength.

B.2
Analysis of 100 tests indicated that 11 specimens failed at 85 MPa or less, and 53 failed at a stress of 100 MPa or less. Estimate the stress that would cause one failure in 10 000.

Proceed from the Weibull equation: $P_s = \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^m\right]$, with $P_s$ denoting the probability that material will not break under stress $\sigma$ (survival probability), $\sigma$ is stress, $\sigma_0$ is a normalising (scale) strength, which may for our purposes be taken as the most probable strength expected ($\sigma_0$ is the value of $\sigma$ at which $P_s = 1/e = 36.78\%$), $m$ is a constant known as the Weibull modulus.

Part C – Manufacturing methods

C 1.
   a) Name five common mould materials for PMCs manufacturing.
b) An important issue in selecting a moulding material is the thermal expansivity of the material. Explain the reason.

c) Explain why heat transfer is of importance in composite manufacturing.

C 2.

a) From composite manufacturing viewpoint, what is the main advantage of thermosets compared to thermoplastics?

b) From the manufacturing and environmental viewpoints, what is the main disadvantage of thermosets compared to thermoplastics?

C 3.

a) Which is the simplest and most versatile of all PMC manufacturing technique?

b) What is the objective of using gel coats?

c) Describe the wet hand lay-up process.

d) Name some components that are usually manufactured by the wet hand lay-up process.

e) Compare viscosities needed for wet hand lay-up and wet spray-up.

C 4.

a) What is a prepreg?

d) Name some components that are usually manufactured from prepregs.

C 5.

a) Name the most common liquid moulding techniques for manufacturing of fibre reinforced composites.

b) Name some advantages with RTM.

c) Name some components that are usually manufactured by the RTM process.

d) Describe the main difference between RTM and Vacuum Injection Moulding (VIM).

e) What is the purpose of carrier layers in the VIM process?

f) Mention main advantages and disadvantages of VIM.

g) Name some different components that are usually manufactured by the VIM process.

C 6.

a) Describe the difference between RIM, RRIM and SRIM.

b) Why are the RIM and compression moulding processes attractive for the automotive industry?
C 7.
   a) What do the abbreviations SMC, BMC, and DMC mean?
   b) Name some components that are usually manufactured by compression moulding.

C 8.
   a) Describe the filament winding process.
   b) Name some components that are usually manufactured by filament winding.

C 9.
   a) Which is the only (common) continuous composite manufacturing process?

C 10.
   a) Describe the differences between thermoset injection moulding and thermoplastic injection moulding.

C 11.
   a) What does the abbreviation GMT mean?
   b) Name an advantage, and a disadvantage of GMTs compared to SMCs.
   c) Describe a commingled prepreg.

Part D – Elastic and thermal properties

D 1.
Prove that the Halpin-Tsai equation for the case $\xi \to \infty$, that is for the case of very long fibres, yields the rule of mixtures, ROM.

D 2.
For a continuous and oriented fiber-reinforced composite, the moduli of elasticity in the longitudinal and transverse directions are 19.7 and 3.66 GPa, respectively. If the volume fraction of fibers is 0.25, determine the moduli of elasticity of fiber and matrix. Answer: proceed from the rules of mixtures, 70.4 and 2.8 GPa, respectively.

D 3.
A composite material consists of polypropylene (PP) with 50 % by weight CaCO$_3$ (calcium carbonate). PP density if 0.9 g/cm$^3$ and CaCO$_3$ is 2.7 g/cm$^3$. CaCO$_3$ particulates are spherical of 1 $\mu$m diameter. Modulus of elasticity of PP is 2 GPa and CaCO$_3$ is 35 Gpa. Assume maximum packing to be 0.62 and proceeding from the Nielsen eq. calculate the composite modulus of elasticity.

D 4.
Proceed from the Halpin-Tsai equation and calculate the Young’s moduli and the major Poisson’s ratio: $E_{11}$, $E_{22}$, $E_{12}$ and $\nu_{12}$ for a unidirectional lamina with constituent fibre and matrix as follows:
<table>
<thead>
<tr>
<th>Property</th>
<th>Fibre</th>
<th>Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus</td>
<td>$E_f = 70$ GPa</td>
<td>$E_m = 3.5$ GPa</td>
</tr>
<tr>
<td>Modulus of rigidity</td>
<td>$G_f = 28$ GPa</td>
<td>$G_m = 1.5$ GPa</td>
</tr>
<tr>
<td>Density</td>
<td>$\rho_f = 2.5$ g/cm³</td>
<td>$\rho_m = 1$ g/cm³</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$\nu_f = 0.22$</td>
<td>$\nu_m = 0.35$</td>
</tr>
<tr>
<td>Weight fraction</td>
<td>$w_f = 0.45$</td>
<td></td>
</tr>
<tr>
<td>Aspect ratio</td>
<td>= 200 (circular fibres, isotropic)</td>
<td></td>
</tr>
</tbody>
</table>

**D. 5**

For some E-glass fiber (tensile strength=3.45 GPa) and epoxy matrix combination, the ratio of critical fiber length to fiber diameter is 50. Determine the fiber-matrix interfacial shear strength.

**D 6.**

An injection-moulded bar contains 20 % (by volume) of short carbon fibres (high strength form) in a matrix of nylon 6.6. A tensile strain of $10^{-4}$ is applied along the axis of the bar. Determine the mean tensile stress carried by the fibres and the overall stress carried by the bar. Assume that the fibres are all of length 400 µm and diameter 6 µm, and are perfectly aligned along the axis of the bar, are perfectly bonded to the matrix, and form a perfect square pattern. Take the axial tensile modulus of the carbon fibres to be 230 GPa, and the tensile and shear moduli of nylon 6.6 to be 2.7 GPa and 1.015 GPa, respectively.

**D 7.**

A thin lamina of a composite with fibres aligned at 45° to the lamina major axis is subjected to the following stress system along its geometric (loading) axes:

\[
\begin{bmatrix} 
\sigma_x \\
\sigma_y \\
\sigma_z 
\end{bmatrix} = \begin{bmatrix} 
10 \\
2 \\
3 
\end{bmatrix} \text{MPa.}
\]

Compute the stress components along the material (principal) axes (i.e., $\sigma_1, \sigma_2,$ and $\sigma_3$).

**D 8.**

A tensile stress of 15 MPa is to be applied to the composite material in a direction transverse to the fibres. Predict the strain that will result in the direction parallel to the fibres. Assume: 40% (by volume) continuous, aligned, glass fibres in a thermoset polyester: $E_f = 76$ GPa, $\nu_f = 0.22$, $E_m = 3.5$ GPa, $\nu_m = 0.38$.

**D 9.**

Strain gage measurements on a deformed laminate show that $\varepsilon_x = 3.5 \times 10^{-3}$, $\varepsilon_y = 5 \times 10^{-4}$, $\varepsilon_z = 5.2 \times 10^{-3}$. The prepreg manufacturer quotes the following as typical data for unidirectional laminate manufactured from his material: $E_{11} = 138$ GN m⁻², $E_{22} = 13.8$ GN m⁻², $G_{12} = 6.9$ GN m⁻², $\nu_{12} = 0.2$. What state of stress is experienced by a 30° ply? What is this state of stress referred to the laminate axes?

**D 10.**
A composite material consists of 60 % (by volume) continuous, uniaxially aligned glass fibres in a matrix of epoxy. Take the tensile modulus and Poisson’s ratio of glass to be 76 GPa and 0.22, and that of epoxy to be 2.4 GPa and 0.34, respectively. A tensile stress of 150 Mpa is applied in a direction inclined at 30° to the fibres. Calculate the tensile strain which results parallel to the fibres.

**D 11.**
An orthotropic lamina has the following characteristics: $E_{11} = 210$ GPa, $E_{22} = 8$ GPa, $G_{12} = 5$ GPa, and $\nu_{12} = 0.3$. Consider a two-ply laminate made of such laminae arranged at $\theta = \pm 60^\circ$ (ply 1 has $\theta = +60^\circ$ and ply 2 $\theta = -60^\circ$) has. Assume that laminae have the same thickness. Compute the matrices $[A]$, $[B]$, and $[D]$. Note that the laminate is antisymmetric.

**D 12.**
A two-ply laminate composite has the top and bottom ply orientations of 45° and 0° and thicknesses of 2 and 4 mm, respectively. The stiffness matrix for the 0° ply is

$$
\begin{bmatrix}
20 & 1 & 0 \\
1 & 3 & 0 \\
0 & 0 & 1 \\
\end{bmatrix} \text{ GPa}
$$

Find the $[\Omega_{ij}]_{45}$ and then compute the matrices $[A]$, $[B]$, and $[D]$ for this laminate.

**D 13.**
Determine the off-axis elastic constants $E_{xx}$, $E_{yy}$, $\nu_{xy}$ and $G_{xy}$ for a lamina containing 60 % vol. of continuous carbon fibres in an epoxy matrix. Assume: $E_f = 220$ GPa, $\nu_f = 0.2$ for the fibre and $E_m = 3.6$ GPa, $\nu_m = 0.35$ for the matrix. Assume that the fibre axis-x is rotated with respect to load direction by $\theta = 45^\circ$. Calculate the $[\Omega]$ matrix.

**D 14.**
Determine the extensional stiffness matrix $[A]$, the coupling matrix $[B]$ and the bending stiffness matrix $[D]$ for:

1) $[-45/+45]$ angle-ply laminate

2) $[+45/-45]$, angle-ply symmetric laminate.

Each lamina is 6 mm thick and contains f = 0.6 carbon fibres in epoxy resin (as in previous problem 13a – use the same material data).
D 15.
Calculate lamina stresses in the \([-45/+45]\) laminate of Problem 14 (part 1), due to a tensile (resultant) load \(N_x = 100 \text{ kN/m}\).

D 16.
Proceed from the constitutive equation for an orthotropic (special) lamina, assume:

\[ E_{11} = 40 \text{ GPa}, \ E_{22} = 10 \text{ GPa}, \ \nu_{12} = 0.28, \ G_{12} = 5 \text{ GPa} \]

and calculate the \([Q_1]\) stiffness matrix. Next consider a \([0^\circ/90^\circ]\) laminate consisting of 3 above specified laminae but of two different thicknesses as shown in the figure below.

![Diagram showing laminate structure with different thicknesses and orientations.](image)

and calculate:

- stiffness matrices \([Q_j]\) for the three laminae
- extensional stiffness matrix \([A]\), coupling stiffness matrix \([B]\), and bending stiffness matrix \([D]\) for the laminate.
D 17.
Assume that the laminate of problem 16 has been loaded with the following (resultant) loads:

\[ N_x = 800 \text{ kN/m} \]
\[ N_y = -550 \text{ kN/m} \]
\[ N_s = 200 \text{ kN/m} \]

Calculate midplane strains and curvatures, laminae stresses.

D 18.
The stiffness coefficients for T300 carbon fibre/5208 epoxy plies are
\[ Q_{11} = 181.8 \text{ GN m}^{-2}, \]
\[ Q_{22} = 10.34 \text{ GN m}^{-2}, \]
\[ Q_{12} = 2.897 \text{ GN m}^{-2}, \]
\[ Q_{66} = 7.17 \text{ GN m}^{-2}. \]

Consider a laminate with the stacking sequence \[ 0^\circ/90^\circ/45^\circ/-45^\circ/0^\circ/-45^\circ/45^\circ/90^\circ \]. The thickness of each ply is 0.125 mm. For uniaxial load applied parallel to the fibres in the \(0^\circ\) plies, the mid-plane strains are
\[ \varepsilon_x^o = 0.98 \times 10^{-5}, \]
\[ \varepsilon_y^o = -0.30 \times 10^{-5}, \]
\[ \varepsilon_s^o = 0.17 \times 10^{-6}, \]
and the principal curvatures are
\[ K_x = 0.91 \times 10^{-2} \text{ m}^{-1}, \]
\[ K_y = -0.14 \times 10^{-2} \text{ m}^{-1}, \]
\[ K_s = -0.20 \times 10^{-2} \text{ m}^{-1}. \]
Determine the magnitude of the uniaxial load.

D 19.
Consider a \([0/90]\) laminate (see figure below) subject to a moisture content linearly varying from a value of 0.5% at the outer surface (most positive \(z\), exposed to the water) to 0% at the inner surface. Note that while the laminate configuration is symmetric, the moisture content distribution is not so. Assume the coefficients of hygroscopic expansion: \( \beta_1 = 0.01 \) and \( \beta_2 = 0.30 \). Each ply is 0.125 mm thick and the elastic properties are:
\[ E_1 = 140 \text{ kN/mm}^2, \]
\[ E_2 = 10 \text{ kN/mm}^2, \]
\[ G_{12} = 5 \text{ kN/mm}^2, \]
\[ \nu_{12} = 0.3 \]
Note that since the laminate configuration is symmetric then all the coupling terms \([B] = [0]\).

Practical notes towards yacht design:

- due to the non-symmetrical distribution of moisture content, the present laminate will warp
- a common misconception designers sometimes make is to assume that a constant moisture content distribution gives the worst case and thus there is no need to consider a non-constant case (as analysed here). As was mentioned in the compendium, the moisture content distribution becomes constant only after a very long period of time (several years). Also, a constant moisture distribution not necessarily gives the worst case. Thus the present case is of prime importance towards yacht design.

Part F – Fracture, toughness, fatigue, creep, damping, lightweight and water absorption performances

F. 1
Compute the longitudinal strength of an aligned carbon fiber-epoxy matrix composite having a 0.25 volume fraction of fibers, assuming the following:

1. an average fiber diameter of 10 micrometers
2. an average fiber length of 5 mm
3. a fiber fracture strength of 2.5 GPa
4. a fiber matrix bond strength of 80 MPa
5. a matrix stress at fiber failure of 10 MPa
6. a matrix tensile strength of 75 MPa

Hint: treat this design problem after studying part D.

F. 2
Compute the longitudinal tensile strength of an aligned continuous glass fiber-epoxy matrix composite in which the average fiber diameter and length are 0.010 mm and 2.5 mm, respectively, and the volume fraction of fibers is 0.40. Assume that

1. the fiber-matrix bond strength is 75 MPa
2. the fracture strength of the fibers is 3500 MPa
3. the matrix stress at fiber failure is 8 MPa.

Hint: treat this design problem after studying part E. Answer: 1340 MPa

F. 3.
Plot a graph of the off-axis fracture strength of a unidirectional continuous fibre carbon/epoxy lamina vs. off-axis angle. Proceed from the Maximum Stress Theory.
Assume:
\[ \sigma_1^* = 700 \text{ MPa}, \quad \sigma_2^* = 22 \text{ MPa}, \quad \sigma_6^* = 60 \text{ MPa} \]

**F 4.**
A high strength carbon/epoxy UD-ply with properties given below is subjected to loading as shown:

![Diagram of ply with loading](image)

The ply properties are:

\[
E_{11} = 140 \frac{\text{kN}}{\text{mm}^2}, \quad E_{22} = 10, \quad G_{12} = 5, \quad \nu_{12} = 0.3
\]

\[
\sigma_1^* = 1500 \frac{\text{N}}{\text{mm}^2}, \quad \sigma_{1c}^* = 1200, \quad \sigma_2^* = 50, \quad \sigma_{2c}^* = 250, \quad \sigma_6^* = 70
\]

\[
\varepsilon_1^* = 1.05\%, \quad \varepsilon_{1c}^* = 0.85, \quad \varepsilon_2^* = 0.5, \quad \varepsilon_{2c}^* = 2.5, \quad \varepsilon_6^* = 1.4
\]

Calculate the failure index (FI) of the ply, using the following non-interactive and interactive theories:

- **Maximum Stress Theory**  \(\text{non-interactive}\)
- **Maximum Strain Theory**
- **Tsai-Hill (maximum work) Theory**  \(\text{interactive}\)

**F 5.**
Proceed from the Tsai-Hill strength criterion and prove that the cross-ply laminate shown in figure below will stand the loads of: \(N_x = 800\ \text{kN/m}, \ N_y = -5.5\ \text{kN/m}, \ N_z = 200\ \text{kN/m}\.\)
It has been calculated (elsewhere, using CLT) that stresses are:

- upper and lower lamina

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_{y_{13}}
\end{bmatrix} = \begin{bmatrix}
206.0 \\
-5.5 \\
19.9
\end{bmatrix} \text{ MPa}
\]

- middle lamina

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_{y_{2}}
\end{bmatrix} = \begin{bmatrix}
48.0 \\
-67.0 \\
19.9
\end{bmatrix} \text{ MPa}
\]

Strengths of the laminae are given

\[
\begin{align*}
\sigma_1^* &= 900 \text{ MPa} \\
\sigma_1^{c*} &= 400 \text{ MPa} \quad \text{(compressive)} \\
\sigma_2^* &= 75 \text{ MPa} \\
\sigma_2^{c*} &= 100 \text{ MPa} \quad \text{(compressive)} \\
\sigma_6^* &= 50 \text{ MPa}
\end{align*}
\]

F 6. A [90/0/90]_s laminate with ply thickness 0.25 mm is subjected to tensile uniaxial loading along the x-direction. Using the Maximum Strain Criterion, find the loads corresponding to first ply failure and subsequent ply failures: then plot the load-strain curve up to failure.

Given:

Failure strains \( \varepsilon_L^* = 0.0105 \) (longitudinal)

\( \varepsilon_T^* = 0.0054 \) (transverse)
The data in the following table on force to fracture, $F_c$, and yield point, $\sigma_y$, are given for CT testing (see figure following the table):

<table>
<thead>
<tr>
<th>material</th>
<th>$F_c$ (N)</th>
<th>$\sigma_y$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A  Epoxy</td>
<td>16.4</td>
<td>100</td>
</tr>
<tr>
<td>B  Toughened Epoxy</td>
<td>63</td>
<td>65</td>
</tr>
<tr>
<td>C  Dry PA 6.6</td>
<td>85</td>
<td>75</td>
</tr>
<tr>
<td>D  Water saturated PA 6.6</td>
<td>210</td>
<td>35</td>
</tr>
</tbody>
</table>

The $c$, $W$, $b$ values are: $c=5\text{mm}$, $W=10\text{mm}$ and $B=6\text{mm}$. $Y$ is given as in eq. F.40 (compendium, valid for anisotropic and isotropic materials). Calculate $K_{IC}$ for each material, and determine which of the tests give valid plane-strain results.

A specially orthotropic, linear viscoelastic composite lamina is subjected to the shear stress-time history shown in figure below. If the effective shear compliance is given by

$$S_{66}(t) = A + Bt \quad \text{when } t \geq 0$$

and

$$S_{66}(t) = 0 \quad \text{when } t < 0$$

where $A$ and $B$ are material constants and $t$ is time, find the expressions for the creep strain at $t < T_0$ and $t > T_0$. 

$$\begin{bmatrix} G \end{bmatrix}_g = \begin{bmatrix} 138.8 & 2.72 & 0 \\ 2.72 & 9.05 & 0 \\ 0 & 0 & 6.9 \end{bmatrix} \text{ GPa}$$
The constituent materials in a unidirectional graphite/epoxy material have the following dynamic mechanical properties at a certain frequency, \( \varphi \):

\[
E'_{f1} = 220 \text{ GPa}, \quad \eta_{f1} = 0.002, \quad \nu_f = 0.6
\]

\[
E'_{m} = 3.45 \text{ GPa}, \quad \eta_{m} = 0.02, \quad \nu_m = 0.4
\]

where \( E \), \( \eta \) and \( \nu \) are modulus, loss factor and volume fraction, respectively. Determine the composite system longitudinal loss factor and the percentage of the total longitudinal damping due to each constituent.